

An approximate analytical solution is obtained for calculating the pressure drop in the flows of a boiling two-phase liquid in a heated channel. The dependence of the maximum temperature in the channel on the rate of flow of the cryogenic fluid is determined.

The possibility of cryostatting superconducting magnetic systems by circulating a two-phase helium flow was examined earlier in sufficient detail in [1, 2]. Such systems are characterized by cooling channels of small diameter (2-10 mm) and great length (100-500 m). The pressure drop in the liquid helium flow is commensurate with the absolute pressure, and the liquid boils as it moves along the channel due to the reduction in absolute pressure over the channel length. Under steady-state conditions, the unit heat flux is generally insignificant, and heat may be removed from the wall to the liquid through convection with a temperature difference of the order of 0.01°K.

There is presently a lack of both theoretical and reliable experimental data on this question. The aim of the present work is therefore to qualitatively analyze the thermal and hydraulic characteristics of a channel such as described above and to approximately evaluate the fluid resistance and other parameters of the flow.

The problem can be set up in the following manner (Fig. 1). An unheated single-phase liquid enters a horizontal, uniformly heated channel. The liquid is heated as it moves and its pressure decreases. The liquid begins to boil when its temperature reaches the saturation temperature in a certain cross section of the channel. The flow in the channel is two-phase after this point, and the mass content of the vapor phase increases as a result of both heating and the pressure decrease. It is necessary for us to find the fluid resistance of the channel, i.e., the dependence of the pressure drop on the flow rate of the liquid, and the maximum temperature of the liquid along the channel. This temperature determines the value of the critical current in the magnet superconductor.

In arriving at a solution, we will make the following assumptions: the liquid flow is homogeneous in the two-phase section; the thermophysical properties of the liquid and its vapor are constant; the liquid is incompressible; the pressure drop due to acceleration of the flow can be ignored due to the relatively low velocity of the liquid; the coefficient of heat transfer from the wall to the liquid is infinitely large, i.e., the temperatures of the wall and liquid are the same (this is valid under the condition that $\alpha F/Gfc_p \gg 1$, as was shown in [3], for example); the thermal load is independent of the wall temperature; the enthalpy of the liquid on the saturation line is independent of the pressure;

$$i' = a + bP_s. \quad (1)$$

The last-mentioned assumption follows from the Clausius-Clapeyron law. If the coefficients a and b are calculated by a linear approximation of the actual properties, then Eq. (1) is not only physically valid, but can be used for quantitative calculations — particularly for helium $a = 3.8 \cdot 10^3$ J/kg, $b = 5.9 \cdot 10^{-1}$ (J·m²)/(kg·N).

Let us first examine the flow of the liquid in the two-phase section, assuming here that the pressure of the channel outlet is known and remains constant. We will simplify the solution by directing the axis counter to the flow and placing the origin at the end of the channel (Fig. 1). In this case, the equation of momentum has the form

$$\frac{dP}{dz} = \xi G^2 v' \left[1 + x \left(\frac{v''}{v'} - 1 \right) \right], \quad (2)$$

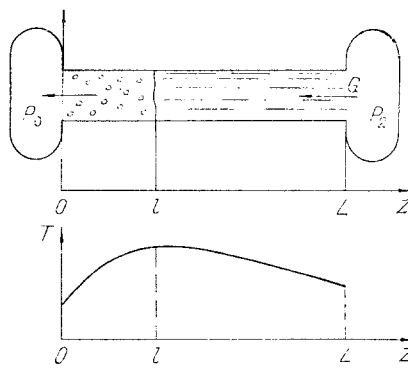


Fig. 1. Diagram of channel and temperature distribution along it in the case of a boiling flow.

where $\xi = \lambda/2d$.

The energy equation is

$$Gf \frac{di_m}{dz} = q. \quad (3)$$

On the basis of (3) and (1), we can determine the mass content of the vapor phase for the boiling liquid, externally heated, in any cross section of the channel:

$$x = \frac{i'_1 - a}{r} - \frac{b}{r} P + \frac{ql(1-z/l)}{Gfr}. \quad (4)$$

Substituting the value of x into (2) and differentiating it with respect to z , we obtain an inhomogeneous second-order linear equation

$$\frac{d^2P}{dz^2} + \xi v' \beta G^2 \frac{dP}{dz} = -\xi v' G^2 \frac{qn}{Gfr}; \quad (5)$$

$$\beta = \frac{bn}{r}; \quad n = \frac{v'' - v'}{v'}.$$

Using the boundary conditions $z = 0, P = P_0$, and $x = x_0$ and considering that the vapor content at the channel outlet can be determined from the heat balance, we obtain a relation for calculating the pressure in any channel cross section in the two-phase section

$$P - P_0 = \left(\frac{qL}{mbGf} + \frac{1 + x_0 n}{\beta} \right) \left[1 - \exp\left(-\frac{mz}{L}\right) \right] - \frac{qz}{Gfb}, \quad (6)$$

where $x_0 = qLs/Gfr$; $s = 1 + (i'_0 - i_2)Gf/qL$; $m = \xi \beta v' G^2 L$.

If we assume that the enthalpy of the liquid on the saturation line is independent of the pressure, then

$$i'_0 - i_2 \approx c_p (T_{s0} - T_2),$$

i.e., this term represents the amount by which the liquid is underheated at the channel inlet relative to the outlet pressure P_0 . The pressure drop occurs completely within the two-phase section when $z = l$ and with allowance for Eq. (1) is equal to

$$\Delta P_f = \frac{i'_1 - i'_0}{b}. \quad (7)$$

In order to determine the length of the two-phase section and the enthalpy of the liquid at the end of the economizer section, let us examine the heated channel as a whole.

Using Eqs. (6) and (7) with $z = l$, we determine the length of the two-phase section from the heat balance on the economizer section

$$\frac{l}{L} = \frac{1}{m} \ln \left(1 + \frac{x_0 n}{1 + \frac{x_0 n}{m}} \right). \quad (8)$$

The total fluid resistance of the channel can be represented as the sum of the resistances in the single- and two-phase sections.

If we consider the local fluid resistances in the economizer section, then, using Eqs. (6), (7), and (8) and the adopted notation, we have

$$\Delta P = \left(\psi \frac{m}{\beta} + \frac{sqL}{Gfb} \right) \left[1 - \frac{1}{m} \ln \left(1 + \frac{x_0 n}{1 + \frac{x_0 n}{m}} \right) \right], \quad (9)$$

where

$$\psi = 1 + \varphi \frac{2d}{\lambda(L-l)}.$$

It is clear from an analysis of (9) that

$$\lim_{G \rightarrow 0} \Delta P = 0; \quad \lim_{G \rightarrow \infty} \Delta P = \psi \xi v' G^2 L,$$

i.e., as the flow rate increases, the pressure drop approaches the fluid resistance in the single-phase flow of the liquid.

The temperature of the liquid in helium circulation systems is generally the same at the inlet and outlet and the value of $s \cong 1$. Considering that $n \cong 6.4$ for liquid helium and that the vapor content at the channel outlet does not exceed 0.4, then for relatively low helium flow rates ($m \leq 1$) we can expand Eq. (8) into a series and, limiting ourselves to the first term, determine the length of the two-phase section:

$$\frac{l}{L} \cong \frac{2}{m+2} \quad (10)$$

and

$$\Delta P \approx \left(\psi \frac{m}{\beta} + \frac{qL}{Gfb} \right) \frac{m}{m+2}. \quad (11)$$

Analysis of Eq. (9) shows that the fluid resistance of a uniformly heated horizontal channel enclosing a flow of boiling helium is a monotonic function and is therefore statically stable [4]. A similar conclusion was reached for the flow of two-phase helium in "short" channels (without allowance for boiling) in [5].

As indicated above, the working current in the superconductor depends to a large degree on the temperature of the cryogenic fluid. It is apparent from the temperature distribution along the channel (Fig. 1) that the liquid is at its highest temperature at the end of the economizer section. We will henceforth refer to this temperature as the "effective" temperature of the cryogen. The effective temperature can be determined from the enthalpy calculated from the heat balance on the economizer section:

$$i'_1 = i_2 + x_0 r \left(1 - \frac{l}{L} \right). \quad (12)$$

Analysis of Eq. (12) shows that an increase in the flow rate of the cryogen leads to a reduction in the vapor content at the outlet on the one hand and, on the other hand, to a reduction in the length of the two-phase section, i.e., the dependence of the maximum temperature of the cryogen on the flow rate is ambiguous. The maximum value of the effective temperature is determined from the condition

$$\frac{di'_1}{dG} = 0$$

and

$$T_{h \max} \approx T_2 + \frac{1}{2} \frac{x_0 r}{c_p} \quad (13)$$

with $m = 2$.

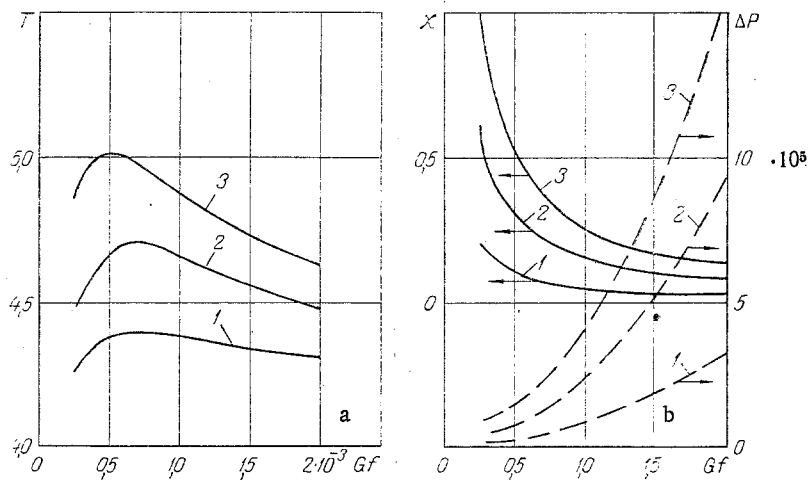


Fig. 2. Effective temperature (a), mass vapor content and pressure drop (b) vs mass flow rate of cryoagent: 1, 2, 3) channel length 100, 300 and 500 m, respectively; T , °K; G_f , kg/sec (a) and 10^{-3} kg/sec (b); ΔP , N/m^2 .

It should be noted that the estimates obtained with Eq. (13) are understated. Nevertheless, the maximum increase in temperature may amount to 0.6–0.8°K.

Figure 2 shows some results of calculation of the effective temperature from Eqs. (12) and (8) and the vapor content at the outlet and pressure drop for a helium flow with the parameters $T_2 = 4.2^\circ K$, $P_0 = 1 \cdot 10^5$ Pa, $d = 0.003$ m, $\lambda = 0.03$, $q = 0.01$ W/m, $L = 100, 300$ and 500 m.

It is apparent from Fig. 2 that the effective temperature is maximal at certain flow rates. Meanwhile, the shorter the channel, the less pronounced the maximum. The maximum effective temperature changes from $4.35^\circ K$ to $5^\circ K$ with a change in channel length from 100 to 500 m. An increase in the flow rate above the value corresponding to the extreme value of effective temperature is relatively ineffective in lowering the effective temperature, but it does lead to a substantial increase in fluid resistance. Given a fixed channel length, to lower the effective temperature it is best to operate in a region of relatively low flow rates and high values of vapor content at the channel outlet, i.e., with a short economizer section. This naturally somewhat impairs heat transfer in the high-vapor-content region, which in turn adversely affects the conditions for cryostatic stabilization of the superconductor. The most radical means of reducing the effective temperature is shortening the channel, but this complicates the structure of the magnet.

NOTATION

c_p , isobaric heat capacity; d , diameter; F , lateral surface of channel; f , surface of channel cross section; G , mass velocity of liquid; q , external heat flux to a unit of the channel length; i , enthalpy (i' — enthalpy of the liquid on the saturation line); L , channel length; l , length of boiling section; P , pressure; ΔP , pressure drop; ΔP_f , pressure drop in the boiling section; r , heat of vaporization; T , temperature; T_h , "effective" temperature; v' , specific volume of liquid; v'' , specific volume of saturated vapor; x , mass content of vapor; z , coordinate; λ , friction coefficient; φ , coefficient of local fluid resistance in the single-phase section. Indices: 0, 2, parameters corresponding to the outlet and inlet cross sections of the channel; 1, parameters corresponding to the beginning of boiling of the liquid; s, equilibrium parameters.

LITERATURE CITED

1. V. E. Keilin and E. Yu. Klimenko, "Cryogenic problems of forced cooled superconducting systems," *Cryogenics*, 12, No. 4, 292–296 (1972).
2. M. Morpurgo, "A large superconducting dipole cooled by forced circulation of two-phase helium," *Cryogenics*, 19, No. 7, 415–423 (1979).
3. V. G. Pron'ko, "On the effect of heat transfer rate and the parameters of the flow of the cryogen (heat-transfer agent) on the rate of cooling and heating of solids," *Inzh.-Fiz. Zh.*, 26, No. 4, 696–700 (1974).

4. I. I. Morozov and V. A. Gerliga, Stability of Fluidized Reactors [in Russian], Atomizdat, Moscow (1969).
5. W. B. Bald and B. A. Hands, "Cryogenic heat transfer at Oxford," Cryogenics, 14, No. 4, 179-197 (1974).

EXPERIMENTAL INVESTIGATION OF LONGITUDINAL MASS TRANSFER IN
THE FLOW OF POLYOXYETHYLENE SOLUTIONS IN A ROUND PIPE

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A method is described for studying longitudinal mass transfer in the turbulent flow of polyoxyethylene solutions in a pipe using nuclear magnetic resonance effects.

This study of longitudinal mass transfer is a continuation of [1]. The phenomenon of nuclear magnetic resonance (NMR) in a flowing liquid [2] makes it possible to study the hydrodynamics of the flow with magnetic marking. Compared to other methods, use of magnetic labelling has such advantages as lack of contact with the flow, an adequate lifetime, and the possibility of studying flows in thin tubes [3].

The improved Taylor method [5] was used in [4]. Longitudinal mass transfer in a pipe with a flow of water and aqueous solutions of polyoxyethylene was studied by introducing NaCl and recording the distribution of the electrical conductivity of the water along the pipe at a certain distance from the site of the labelling. Data was obtained for different concentrations of polyoxyethylene.

Figure 1 shows a block diagram of the experimental unit for studying longitudinal mass transfer by the NMR method. The pipe 1 is a glass tube 700 mm long with an inside diameter of 4.8 mm. The liquid is magnetized beforehand by passing it through a polarizer 2, where the magnetic moments of the protons in the water molecules were preferentially oriented in the direction of the strong magnetic field.

Located at the end of the tube is a detector 3, where, with the aid of a standard IMI-2 magnetic inductometer, an NMB signal is recorded. The signal strength here is proportional to the total magnetic moment of the protons in a unit volume of the liquid M . The liquid is pumped from a 50-liter tank by a pump 4 placed at the tube outlet. The section of tube investigated was a portion of length $L = 380$ mm located between the nutation coil 5 and the coil of the detector 3.

The molecules were marked by means of the coil 5, which creates a weak variable magnetic field directed along the dynamic velocity of the flow and perpendicular to the vector of the external magnetic field B . The nutation coil is fed from a G4-26 generator, which creates an alternating electric field with a frequency of 100 kHz — equal to the frequency of precession of the nuclei in the field B — and a power corresponding to a 180° rotation of the magnetization vector. The signal is modulated by rectangular pulses with a frequency $\nu = 2-40$ Hz. The NMR signal is observed and the signal strength read from an oscillograph screen.

In studying longitudinal diffusion, liquid particles to one side of a cross section of the stream perpendicular to the flow direction must be marked and the number of marked particles which turn up on the other side of this cross section after a certain period of time has to be determined.

We will assume that all of the particles move at the same velocity v_{av} . After the field of the nutation coil has acted on the liquid, alternating volumes of positive and negative magnetization separated by sharp boundaries appear in the fluid. The length of each volume

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